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Gamma echo in forward nuclear resonant scattering of synchrotron radiation in an inhomogeneous medium

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Abstract. The γ-echo technique in nuclear forward scattering of synchrotron radiation has been suggested. This technique is based on abrupt 180° remagnetization of the sample at $t = \tau$ which creates the γ-echo with a maximum at $t = 2\tau$. For theoretical calculations the Maxwell–Bloch formalism modified for multilevel nuclear states has been employed. The numerical simulation for a medium of any 'optical' thickness as well as for the case with superparamagnetic relaxation has been carried out. It is shown that

(1) successive 180° remagnetization at $t_i = (2i + 1)\tau$ generates the γ-echo signals at $t_{ei} = 2(i + 1)\tau$,
(2) the peak of the γ-echo for an 'optically' thick sample lies above the curve of resonant response for one of the same thickness with no inhomogeneity of hyperfine fields and
(3) superparamagnetic relaxation attenuates the γ-echo signal and changes its form.

1. Introduction

The researching of disordered systems is an actual problem of material science as well as in studying of different protein complexes. Analysis of local atomic environment and of dynamical effects due to the interaction with nearest neighbours gives additional information about the structure of investigated objects and allows us to predict their properties and the application. The usefulness of any spectroscopic method in this field (electron spin resonance, nuclear magnetic resonance, gamma-resonant spectroscopy, neutron scattering) is defined by the ability to distinguish the contributions of both static (structural) and dynamical effects (superparamagnetism of magnets [1], fluctuation processes in the electron shell of a paramagnetic atom), which result in the inhomogeneous and homogeneous broadening of a spectral line, respectively. In order to pick out the inhomogeneous broadening of a spectral line the spin echo technique has been suggested [2] in radiospectroscopy, which was later realized in optics as photon echo [3]. Briefly consider it in the conventional vector model, which describes the behaviour of a system of the nuclear angular momenta ($I_g = 1/2$) in an external magnetic field. This field contains: (1) a constant one which induces the splitting into two sublevels for the ground state of the kth nucleus so that the splitting value ω_k lies in the $\omega_c \pm \Delta\omega_{inh}$ interval ($\Delta\omega_{inh}$ is the frequency spread of ω_k because of the inhomogeneity of the local environment) and (2) the pair of radiofrequency (RF) field pulses of ω_c frequency with Δt_1, Δt_2 time durations and with the delay time $\tau \geqslant 1/\Delta\omega_{inh}$ so that $\Delta t_1, \Delta t_2 \ll \tau$. The first pulse ($\pi/2$-pulse, $\omega^{rf}\Delta t_1 \approx \pi/2$, $\omega^{rf} = -\mu_g H_{rf}/\hbar$) transfers the nuclear spin system from the pure state, which is described by the longitudinal nuclear magnetization M_z, to the

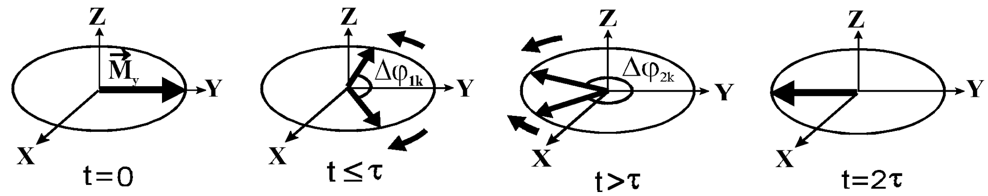

Figure 1. Vector model of primary spin echo.

coherent state, which is described by the transverse nuclear magnetization $\boldsymbol{M}_y = \sum_k \boldsymbol{M}_{ky}$, so that directions of all $\boldsymbol{M}_{ky}$ coincide with each other and $|\boldsymbol{M}_y|$ is of maximum value (figure 1). By freely precessing in the XY plane each vector $\boldsymbol{M}_{ky}$ acquires after dephasing time τ the direction determined by the angle $\Delta\varphi_{1k} = \Delta\omega_{inh}^{k}\tau$, where $\Delta\omega_{inh}^{k} = \omega_k - \omega_c$. As a result the modulus of vector $\boldsymbol{M}_y$ is essentially decreased. In this moment the second pulse (π-pulse, $\omega^{rf}\Delta t_2 \approx \pi$) generates 180° rotation of all $\boldsymbol{M}_k$ around the X axis and, now, their directions are defined by the angles $\Delta\varphi_{2k} = 180^\circ - \Delta\varphi_{1k}$. Through rephasing time τ each angle $\Delta\varphi_{2k}$ receives the incrementation $\Delta\varphi_{1k}$ and directions of all $\boldsymbol{M}_k$ coincide again by making $|\boldsymbol{M}_y|$ of maximum value. This is the essence of the primary spin echo with the peak at $t_e = 2\tau$.

Time domain methods of nuclear resonant spectroscopy and γ-optics are being intensively developed since synchrotron radiation (SR) facilities are coming into operation. Although some echo-like effects of quantum interference have been investigated in [4] and [5], it is impossible to directly observe the classical γ-echo so far, because SR intensity for the operation frequency range ($\sim 10^{20}$–10^{21} Hz) is very low to create $\pi/2$ and π γ-pulses. However, there is an original way out [6] from this situation for soft magnets (metal glasses, alloys, ferrous nuclei of proteins) where iron is one of the main components. In these systems axial hyperfine (HF) interaction gives rise to the splitting of both ground ($E_g = 0$, $I_g = 1/2$) and excited ($E_e = 14.413$ keV, $I_e = 3/2$, $\tau_n \approx 141.1$ ns) states of each resonant nucleus and provides the orientation of local hyperfine fields $H_{hf}^{(k)}(k = 1, \ldots, N)$ along the direction of external magnetic field. In general the HF splitting values $\omega_j^{hf(k)} = -\mu_j \boldsymbol{H}_{hf}^{(k)}/\hbar(j = e, g)$ are considerably more than the full width of the excited state ($\Gamma = 1/\tau_n$, $\sim$7 MHz), but they are greatly less than the value of frequency spread ($\gamma \sim 10^6$ MHz) of the 'instantaneous' ($t_0 \sim 200$ ps time duration) SR pulse incoming into the sample after x-ray monochromatization. Therefore, one can suggest that all resolved nuclear transitions between HF sublevels are excited simultaneously and independently from each other. Their interference creates the pattern of quantum beats of the resonant response (RR) to SR pulse. In each transition, described by the effective two-level model, the transverse current $\langle \boldsymbol{j}\rangle_{eg}^{(k)} = \boldsymbol{j}_{eg}^{(k)}\rho_{ge}^{(k)}(t)$ is induced, where $\boldsymbol{j}_{eg}^{(k)}$, $\rho_{ge}^{(k)}(t)$ are the matrix elements of the current vector-operator and of the density matrix of the kth nucleus, respectively (figure 2). The precession frequency in the XY plane of each current $\langle \boldsymbol{j}\rangle_{eg}^{(k)}$ is $\omega_{eg}^{(k)} = m_e\omega_e^{hf(k)} - m_g\omega_g^{hf(k)}$, which lies within the $\omega_{eg}^{(0)} \pm \Delta\omega_{eg}^{inh}$ interval ($\Delta\omega_{eg}^{inh}$ is the frequency spread due to the inhomogeneity of the local atomic environment). Directions of all $\langle \boldsymbol{j}\rangle_{eg}^{(k)}$ almost coincide with each other at time $\Delta t \ll \tau \geqslant 2\pi/\Delta\omega_{eg}^{inh}$ and, hence, the modulus of the total current $\langle \boldsymbol{j}\rangle_{eg} = \sum_k \langle \boldsymbol{j}\rangle_{eg}^{(k)}$ is of maximum value. When $\Delta t \to \tau$ the directions of all $\langle \boldsymbol{j}\rangle_{eg}^{(k)}$ defined by the angles $\Delta\varphi_{1k} = \Delta\omega_{eg}^{(k)}\tau(\Delta\omega_{eg}^{(k)} = \omega_{eg}^{(k)} - \omega_{eg}^{(0)})$ are essentially distinguished each from other and the modulus of $\langle \boldsymbol{j}\rangle_{eg}$ is considerably reduced. This leads to a speedup of resonant response decay by comparing with that for $\Delta\omega_{eg}^{inh} = 0$. Let the local hyperfine fields $\boldsymbol{H}_{hf}^{(k)}$ be 'instantaneously' reversed at $t_3 = \tau$ owing to the fast

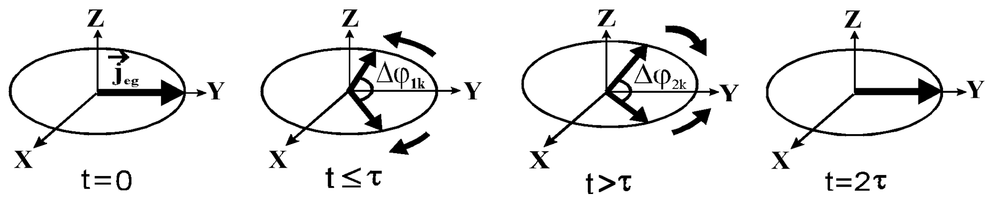

Figure 2. Vector model of the γ-echo.

remagnetization of the sample within a few nanoseconds. Then $\omega_{eg}^{(k)} \to -\omega_{eg}^{(k)}$ and the all currents $\langle \boldsymbol{j}\rangle_{eg}^{(k)}$ will rotate to the opposite side. Their directions coincide again at $t_e = 2\tau$ and this gives rise to a sharp increase of the modulus of $\langle \boldsymbol{j}\rangle_{eg}$. Thus the γ-echo is created in resonant response. Its form will be determined by the envelope of quantum beats centred at $t = t_e$, which depends on the distribution function of HF fields $g(H_{hf})$. It is seen the interference effect, induced by fast switching of the HF field, is of great importance. It was intensively studied in both nuclear diffraction [7] and nuclear forward scattering [8] when there is no distribution of local hyperfine field. In our paper we will use this idea to model the γ-echo in forward nuclear scattering because this channel as well as incoherent nuclear scattering is of great significance for investigations of magnetic structures with no long-range order.

2. Theoretical formalism and analysis of results

To solve the problem we will be using Maxwell–Bloch formalism modified for multilevel structure of nuclear states [9]. The high-temperature approximation is supposed to be right in which sublevels of ground nuclear state are equally populated. First of all we consider the case under no superparamagnetism in a nuclear target. The corresponding equation system in general form, describing the propagation of SR pulse through a resonant medium, can be written as

$$\begin{cases} \dfrac{\partial \boldsymbol{a}}{\partial z} = \dfrac{2\pi \mathrm{i}}{\omega} \displaystyle\sum_{k,e,g} \rho_{ge}^{(k)} \boldsymbol{j}_{eg}^{(k)} \\ \dfrac{\partial \rho_{ge}^{(k)}}{\partial \tau} = -\mathrm{i}\left(\Delta - \mathrm{i}\dfrac{\Gamma}{2}\right)\rho_{ge}^{(k)} - \dfrac{\mathrm{i}}{\hbar}[\hat{H}_{(k)}, \rho^{(k)}]_{ge} \end{cases} \tag{1}$$

where $\boldsymbol{a}(t, z)$ is the vector-potential envelope of the electromagnetic field and Δ the Doppler shift of the incident radiation frequency.

The model Hamiltonian is $\hat{H}_{(k)} = \sum_j \hat{H}_{(k)}^{(j)} + \hat{V}_{(k)}^{(\gamma)}$, where $\hat{V}_{(k)}^{(\gamma)}$ is the general Hamiltonian of electromagnetic interaction with the kth nucleus.

$$\hat{H}_{(k)}^{(j)} = \hat{H}_{(k)hf}^{(j)} + \hat{H}_{(k)Q}^{(j)}.$$

Here $\hat{H}_{(k)hf}^{(j)} = \hbar\omega_j^{hf(k)}(t)\hat{I}_{jz}$ is the Hamiltonian of magnetic HF interaction and

$$\hat{H}_{(k)Q}^{(j)} = \hbar\omega_{jQ}^{(k)}\left(\hat{I}_{jz}^2 - \tfrac{1}{3}I_j(I_j+1) + \frac{\eta}{6}(\hat{I}_{j+}^2 + \hat{I}_{j-}^2)\right)$$

the Hamiltonian of quadrupole interaction.

Usually amorphous soft magnets are of ‘easy plane’ type for which the condition $\omega_e^{hf(k)} \gg \omega_{eQ}^{(k)} \sim \Gamma$ $(\omega_{gQ}^{(k)} = 0)$ is fulfilled. Hence, HF interaction makes a great contribution

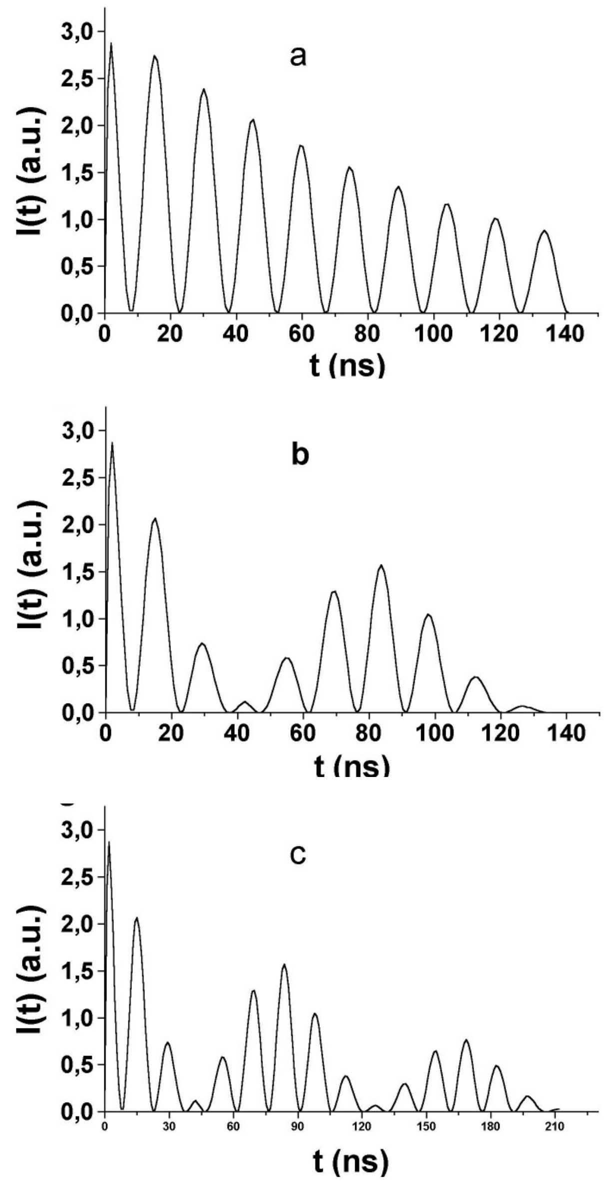

Figure 3. γ-echo signal: π-polarized SR pulse; $b = 0.5$, $g(H_{hf})$ is Gaussian, $H_{hf}^0 \approx 32$ T, (a) $\Delta H_{hf} = 0.0$, (b) $\tau = 42$ ns, $\Delta H_{hf} = 0.5H_{hf}^0$, (c) $\tau_1 = 42$ ns, $\tau_2 = 126$ ns, $\Delta H_{hf} = 0.5H_{hf}^0$.

to inhomogeneous frequency spread for nuclear transitions and Hamiltonian $\hat{H}_{(k)}$ in (1) can be replaced by: $\hat{H}'_{(k)} = \hat{H}_{(k)} - \sum_j \hat{H}^{(j)}_{(k)Q}$.

Let local magnetic HF fields of the sample be oriented along the external magnetic field in the 'easy plane'. Because all matrix elements $\boldsymbol{j}_{eg}^{(k)}$ are of the same form and $N \gg 1$ it is correct to make substitutions in (1): $\rho_{eg}^{(k)} \to \rho_{ge}(H_{hf})$, $\hat{H}'_{(k)} \to \hat{H}'(H_{hf})$,

$$\sum_k \rho_{ge}^{(k)} \boldsymbol{j}_{eg}^{(k)} \to N \int \mathrm{d}H_{hf}\, g(H_{hf}) \rho_{ge}(H_{hf}) \boldsymbol{j}_{eg}.$$

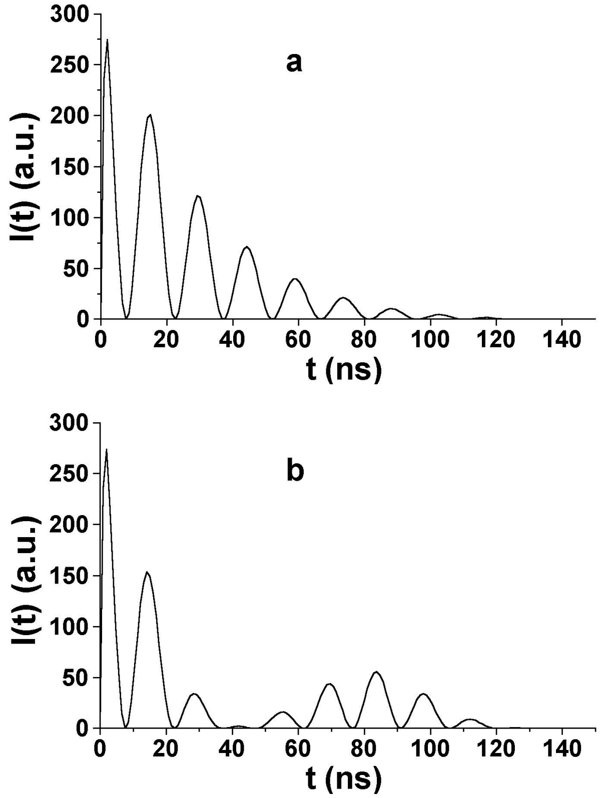

Figure 4. γ-echo signal: π-polarized SR pulse; $b = 5.0$, $H_{hf}^0 \approx 32$ T. (a) Resonant response decay with no inhomogeneity of hyperfine fields. (b) $g(H_{hf})$ is Gaussian ($\Delta H_{hf} = 0.5 H_{hf}^0$), $\tau = 42$ ns.

Now, by using [10], the system (1) can be written for the ^{57}Fe isotope in a circular polarization basis of incident radiation ($p = \pm 1$):

$$\begin{cases} \dfrac{\partial a_p}{\partial z} = -\frac{3}{4}\mu\Gamma \displaystyle\sum_{M,e,g} d_{Mp}^{(1)}(\vartheta) C(\tfrac{1}{2}\, 1\, \tfrac{3}{2} m_g M m_e) \times \int \mathrm{d}H_{hf}\, g(H_{hf}) \rho_{ge} \\ \dfrac{\partial \rho_{ge}}{\partial t} = -\mathrm{i}\left(\Delta - \omega_{eg}(t) - \mathrm{i}\dfrac{\Gamma}{2}\right)\rho_{ge} + \displaystyle\sum_{p'} d_{Mp'}^{(1)}(\vartheta) C(\tfrac{1}{2}\, 1\, \tfrac{3}{2} m_g M m_e) a_{p'} \end{cases} \tag{1a}$$

where μ is the standard coefficient of nuclear resonant absorption, $d_{\cdots}^{\cdots}$ the rotation matrix; $\vartheta = \pi/2$, $\varphi = 0$ are the polar and azimuth angles of the wave vector of incident radiation in the spherical coordinate basis which is defined by the external magnetic field and $C(\ldots)$ the Clebsch–Gordan coefficients.

The functions

$$\omega_{eg}(t) = \begin{cases} \omega_{eg}(H_{hf}) & t \leqslant \tau \\ -\omega_{eg}(H_{hf}) & t > \tau \end{cases}$$

mean that 'instantaneous' reversal of hyperfine field H_{hf} occurs at $t = \tau$. For any value of μl (l is the linear size of the sample) the system ($1a$) is to be numerically solved. To correctly explain the results of numerical simulation let us consider the case of an optically thin sample

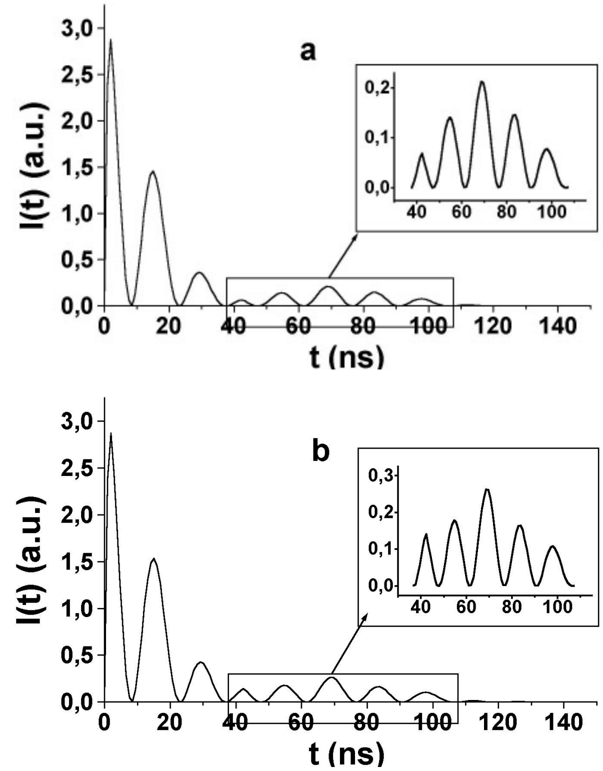

Figure 5. γ-echo signal under superparamagnetic relaxation: π-polarized SR pulse; $b = 0.5$, $\tau = 42$ ns, $g(H_{hf})$ is Gaussian ($H^0_{hf} \approx 32$ T, $\Delta H_{hf} = 0.5H^0_{hf}$), $g(W)$ is Gaussian ($W_0 = 2.0$ MHz). (a) $\Delta W = 0.5$ MHz; (b) $\Delta W = 2.0$ MHz.

($\mu l \ll 1$). Then $a_{p'}(t, z)$ can be replaced in the time equations for ρ_{ge} of (1a) by

$$a^{inc}_{p'}(t) = \sqrt{I_0} c_{p'}\, \mathrm{e}^{-\gamma t/2} \qquad t \gg t_0$$

where I_0 is the incident radiation flux.

Under this assumption the vector-potential envelope of the RR field is calculated in analytical form. It can be made via the formalism of a rotation matrix $d^{(I_j)}_{j_1 j_2}(\beta_j)$ [11], which was used for researching the influence of instantaneous remagnetization the $^{57}FeBO_3$ on coherent scattering of SR [12]. However the direct integration (1a) is easily carried out.

Let $g(H_{hf}) = (1/\sqrt{\pi}\Delta H_{hf})\mathrm{e}^{-((H_{hf}-H^0_{hf})/\Delta H_{hf})^2}$ be a Gaussian of ΔH_{hf} width which is centred at H^0_{hf} and the incident radiation have the linear σ-polarization (those nuclear transitions are excited for which $M = \pm 1$): $c_{p'} = 1/\sqrt{2}$, $a^{inc}_{\sigma}(t) = (a^{inc}_1(t) + a^{inc}_{-1}(t))$.

It follows from (1a) that the resonant response has a linear σ-polarization as well and its intensity at $\Delta = 0$ is of the form

$$I^{res}_{\sigma}(t) \sim |a^{res}_{\sigma}|^2 = I_0 b^2 g^2\, \mathrm{e}^{-\Gamma t}\left(\mathrm{e}^{\frac{-\alpha^2}{4}\omega_1^2 t^2} \cos\omega_1 t + \tfrac{1}{3}\, \mathrm{e}^{\frac{-\alpha^2}{4}\omega_2^2 t^2} \cos\omega_2 t\right)^2 \qquad t \leqslant \tau \tag{2a}$$

$$I^{res}_{\sigma}(t) \sim |a^{res}_{\sigma}|^2 = I_0 b^2 g^2\, \mathrm{e}^{-\Gamma t}\left(\mathrm{e}^{\frac{-\alpha^2}{4}\omega_1^2 (t-2\tau)^2} \cos\omega_1 (t-2\tau) + \tfrac{1}{3}\, \mathrm{e}^{\frac{-\alpha^2}{4}\omega_2^2 (t-2\tau)^2} \cos\omega_2 (t-2\tau)\right)^2 \qquad t > \tau \tag{2b}$$

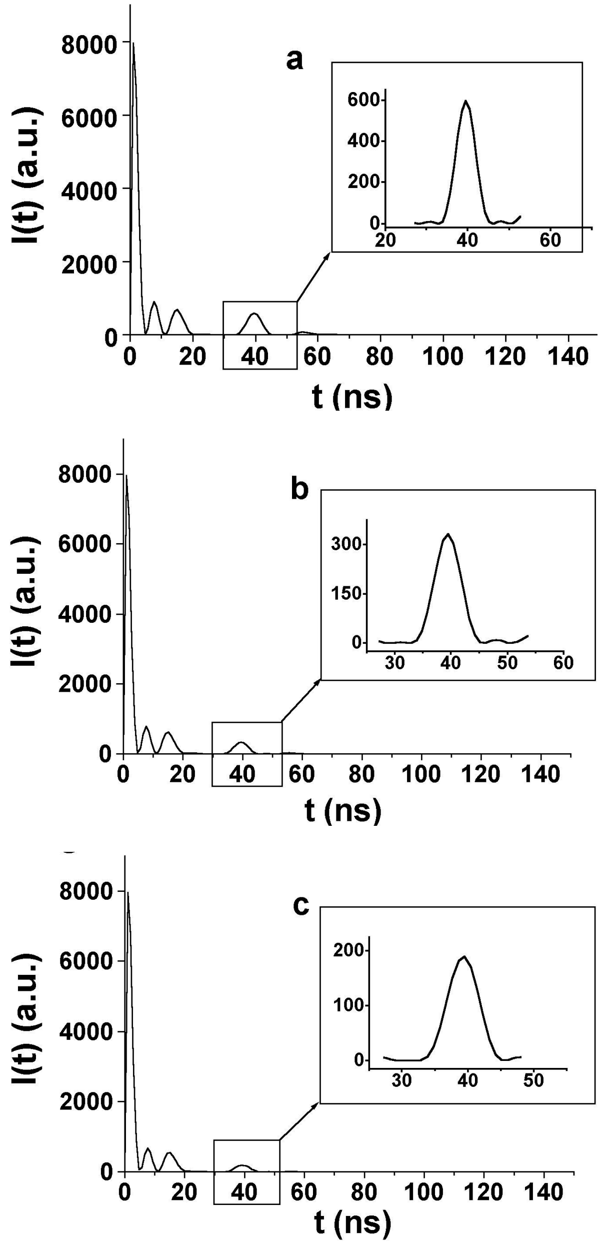

Figure 6. γ-echo signal under superparamagnetic relaxation: σ-polarized SR pulse; $b = 30.0$, $\tau = 20$ ns, $g(H_{hf})$ is Gaussian ($H_{hf}^0 \approx 32$ T, $\Delta H_{hf} = 0.5 H_{hf}^0$), $g(W) = \delta(W - W_0)$. (a) $W_0 = 0.0$ MHz; (b) $W_0 = 1.0$; (c) $W_0 = 2.0$ MHz.

where

$$b = \frac{3}{4}\mu l \ll 1 \qquad g = \frac{\Gamma}{\gamma} \ll 1 \qquad \alpha = \frac{\Delta H_{hf}}{H_{hf}^0} \qquad \omega_1 = -0.5(3\omega_e^{hf}(H_{hf}^0) - \omega_g^{hf}(H_{hf}^0))$$

$$\omega_2 = 0.5(\omega_e^{hf}(H_{hf}^0) + \omega_g^{hf}(H_{hf}^0)).$$

If an incident SR pulse has a linear π-polarization (those nuclear transitions are excited for which $M = 0$): $c_{p'} = p'/\sqrt{2}$, $a_\pi^{inc}(t) = \mathrm{i}(a_1^{inc}(t) - a_{-1}^{inc}(t))$, then like the previous case the resonant response has a linear π-polarization and its intensity at $\Delta = 0$ is of the form:

$$I_\pi^{res}(t) \sim \tfrac{16}{9} I_0 b^2 g^2\, \mathrm{e}^{-\Gamma t}\, \mathrm{e}^{-\frac{\alpha^2}{2}\omega_3^2 t^2} \cos^2 \omega_3 t \qquad t \leqslant \tau \qquad (2c)$$

$$I_\pi^{res}(t) \sim \tfrac{16}{9} I_0 b^2 g^2\, \mathrm{e}^{-\Gamma t}\, \mathrm{e}^{-\frac{\alpha^2}{2}\omega_3^2 (t-2\tau)^2} \cos^2 \omega_3 (t - 2\tau) \qquad t > \tau \qquad (2d)$$

where $\omega_3 = -0.5(\omega_e^{hf}(H_{hf}^0) - \omega_g^{hf}(H_{hf}^0))$.

It is seen that $(2c, d)$ are simpler to understand because the quantum beats are of one frequency only. The analysis of $I_\pi^{res}(t)$ shows (figure 3(a),(b)) the speedup of RR decay takes place at $t_0 \ll t \leqslant \tau$, which is defined now by the time decay $t_d = \sqrt{2}/\alpha\omega_3 \ll \tau_n$. In contrast, the amplitude of the resonant response is increased at $t > \tau$ up to a maximum at $t_e = 2\tau$ by creating a γ-echo signal. Its form is the envelope of quantum beats near t_e. That is a Gaussian with full width at half maximum (FWHM) and it allows to directly define ΔH_{hf}.

If $g(H_{hf}) = (\Delta H_{hf}/2\pi)/[(H_{hf} - H_{hf}^0)^2 + (\Delta H_{hf}/2)^2]$ is Lorentzian, then

$$I_\pi^{res}(t) \propto \tfrac{16}{9} I_0 b^2 g^2\, \mathrm{e}^{-\Gamma t}\, \mathrm{e}^{-\alpha\omega_3 t} \cos^2 \omega_3 t \qquad t \leqslant \tau \qquad (3a)$$

$$I_\pi^{res}(t) \propto \tfrac{16}{9} I_0 b^2 g^2\, \mathrm{e}^{-\Gamma t}\, \mathrm{e}^{\alpha\omega_3 (t-2\tau)} \cos^2 \omega_3 (t - 2\tau) \qquad \tau < t \leqslant 2\tau \qquad (3b)$$

$$I_\pi^{res}(t) \propto \tfrac{16}{9} I_0 b^2 g^2\, \mathrm{e}^{-\Gamma t}\, \mathrm{e}^{-\alpha\omega_3 (t-2\tau)} \cos^2 \omega_3 (t - 2\tau) \qquad t > 2\tau$$

and the γ-echo signal is the Lorentzian with FWHM $2\ln 2/\alpha\omega_3$ centred at $t_e = 2\tau$.

In the general case successive remagnetization of the nuclear target at the moments $t_i = (2i + 1)\tau$ $(i = 0, 1, 2, \ldots)$ gives rise to the condition:

$$\omega_{eg}(t) \begin{cases} \omega_{eg}(H_{hf}) & t \leqslant \tau \\ (-1)^{i+1}\omega_{eg}(H_{hf}) & (2i+1)\tau < t \leqslant (2i+3)\tau \end{cases}$$

and leads to γ-echo signals in resonant response with peaks at moments $t_{ei} = 2(i + 1)\tau$.

For example, if $g(H_{hf})$ is Gaussian, then

$$I_\pi^{res}(t) \sim \tfrac{16}{9} n b^2 g^2\, \mathrm{e}^{-\Gamma t}\, \mathrm{e}^{-\frac{\alpha^2\omega_3^2}{2}(t-2(i+1)\tau)^2} \cos^2 \omega_3 (t - 2(i+1)\tau)$$
$$(2i+1)\tau < t \leqslant (2i+3)\tau \qquad (3c)$$

(figure 3(c)).

At $\mu l \geqslant 1$ the results of numerical simulation of $(1a)$ are in good agreement with analytical expressions (2), (3) in both location and shape of the γ-echo (figure 4(a)). One difference is that echo peaks do not lie on the curve $\mathrm{e}^{-\Gamma t}$ as in the case $\mu l \ll 1$. But they are situated above the corresponding curve of resonant response obtained for $\Delta H_{hf} = 0$ (figure 4(b)). This is the consequence of the competition between dynamical effects in an optically thick sample and the interference once forming an echo signal. The former result in speedup of resonant response and the latter try to bring the maximum of the echo signal to $\mathrm{e}^{-\Gamma t}$. If fluctuations of local magnetic momenta in the nuclear target due to the temperature and dimensional effects (superparamagnetism, area of phase transition ferro/ferri-magnet $\to$ paramagnet in bulk) take place, then system (1) has to modified to the following form:

$$\begin{cases} \dfrac{\partial a_p}{\partial z} = -\dfrac{3}{4}\mu\Gamma \displaystyle\sum_{M,e,g,v} d_{Mp}^{(1)}(\vartheta) C(\tfrac{1}{2}\, 1\, \tfrac{3}{2} m_g M m_e) \int \mathrm{d}H_{hf}\, \mathrm{d}W g(H_{hf}) g(W) \rho_{ge}^{(v)} \\ \dfrac{\partial \rho_{ge}^{(v)}}{\partial t} = \mathrm{i}\left(\Delta - \nu\omega_{eg}(t) + \mathrm{i}\left(\dfrac{\Gamma}{2} + W\right)\right)\rho_{ge}^{(v)} \\ \quad + W\rho_{ge}^{(-v)} + \displaystyle\sum_{p'} d_{Mp'}^{(1)}(\vartheta) C(\tfrac{1}{2}\, 1\, \tfrac{3}{2} m_g M m_e) a_{p'} \end{cases} \qquad (4)$$

where $\nu = \pm 1$, W the superparamagnetic relaxation rate and $g(W)$ the distribution function of W over the sample.

At any W the system (4) has to be numerically solved even for an optically thin sample. At $\mu l \ll 1$ the modelling of γ-echo was made when the nuclear target is an ensemble of superparamagnetic particles and $g(W) = (1/\sqrt{\pi}\Delta W)\,\mathrm{e}^{-((W-W_0)/\Delta W)^2}$ $(W > 0)$ a Gaussian of width ΔW centred at W_0 (figures 5(a),(b)). It is seen that the echo signal grows with increasing ΔW. This happens because the broadening of $g(W)$-wings allows for smaller W to attenuate the total influence of superparamagnetic relaxation. At $\mu l \gg 1$ the modelling was performed when the nuclear target is a bulk magnet in the phase transition area ($g(W) = \delta(W - W_0)$) and the SR-pulse is of σ-polarization. The analysis of figure 6(a)–(c) shows that increasing of W within the interval where HF structure of the nuclear levels is resolved gives rise to attenuation of echo signal. This result corresponds to the observation condition of both classical spin and photon echoes—the time of reversible relaxation must be much less than that of irreversible relaxation [13].

3. Conclusion

We have investigated the gamma echo technique due to 180° remagnetization of a sample in forward nuclear resonant scattering. To observe these echo phenomena in experiment those soft magnets should be selected which allow fast (within a few nanoseconds) switching of local hyperfine fields. It was shown if the sample magnetic momentum 'instantaneously' reverses direction at $t = \tau$ then a peak of the echo signal is observed at $t_e = 2\tau$. Its form is the envelope of quantum beats which is directly defined by the distribution function of local hyperfine fields. Thus it can provide a good possibility to obtain additional information on the nearest environment in inhomogeneous magnetic structures directly from experiment. Such information may be useful, e.g. for studying the properties of magnetic amorphous nuclei in iron-accumulating proteins (ferritin, bacterioferritin) [14]. Successive 180° remagnetization of a nuclear target at $t_i = (2i+1)\tau$ $(i = 0, 1, 2, \ldots)$ leads to arising of echo signals with peaks at $t_{ei} = 2(i+1)\tau$ and it can ensure more reliable information on the local atomic environment. Amorphous magnetic structures are suitable objects for these experiments because the magnetoelastic interaction is considerably weaker and multiple remagnetization of the sample has not led to its strong heating and to loss of the magnetic properties. When $\mu l \geqslant 1$ the peak of γ-echo is more than the corresponding value of resonant response with no inhomogeneous spread of hyperfine field. This property makes the given technique very attractive for investigation of the short-range order in an optically thick sample where resonant response intensity is much greater. The echo amplitude dependence on the superparamagnetic relaxation rate opens an additional possibility for investigation of superparamagnetism in magnets. Among the major applications of the γ-echo technique one may be the studies of paramagnetic ferrous complexes with isotropic HF interaction included in different proteins. If the sample is cooled down to the temperature where the electron relaxation can be neglected then the character of the inhomogeneity of the ligand environment can be analysed. After that the obtained results can be used for studying pure dynamical effects in an electron shell. The last problem is out of the scope of this paper and should be considered separately as well as the γ-echo technique in the incoherent channel of nuclear resonant scattering of SR. Such modelling has to be done because third generation synchrotron facilities permit us to realize the corresponding experiments.

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